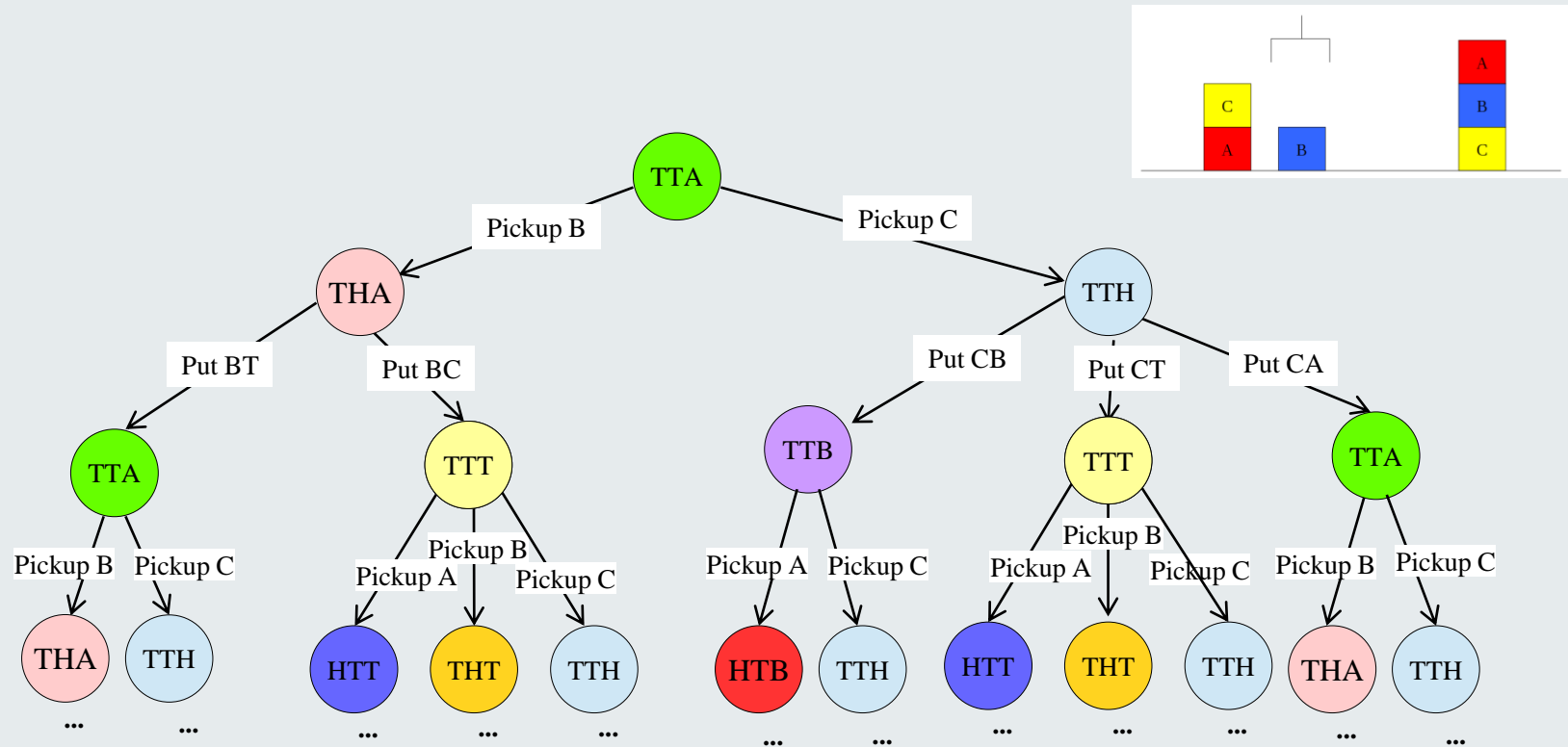




# **Expressive Planning by Combining Forward Search and Mixed-Integer Programming**

Elad Denenberg

# Classical Planning as Forward Search



Classical Planning Gives Us:

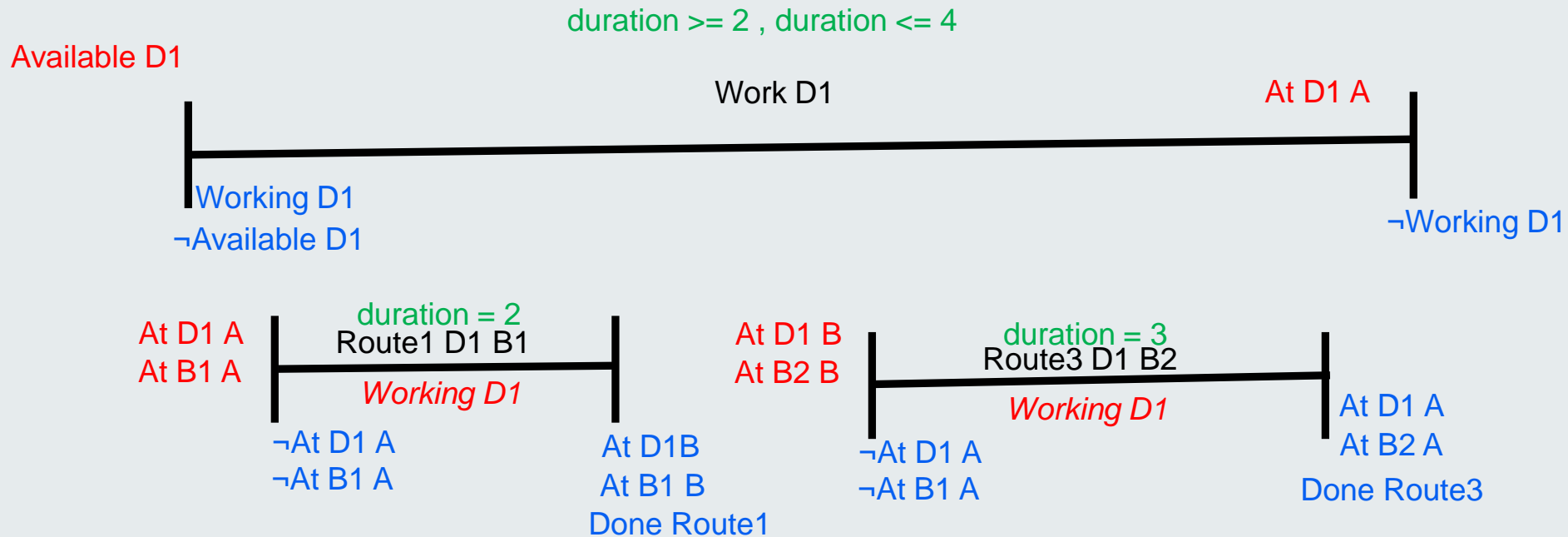
- Propositional Relaxation Heuristics: RPG, Causal Graph;
- Search Guidance: Helpful Actions/Preferred Operators.
- Search Techniques, Enforced Hill-Climbing, Multi Open List Search, Memoisation;

# Running Example – Public Transport

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- Drivers have working hours;
- Bus routes have fixed durations and start and end locations.
- Goals are that each bus route is done.
- The routes have timetables that they must follow.

# Temporal Planning: Public Transport



Conditions and Effects at the start and at the end;

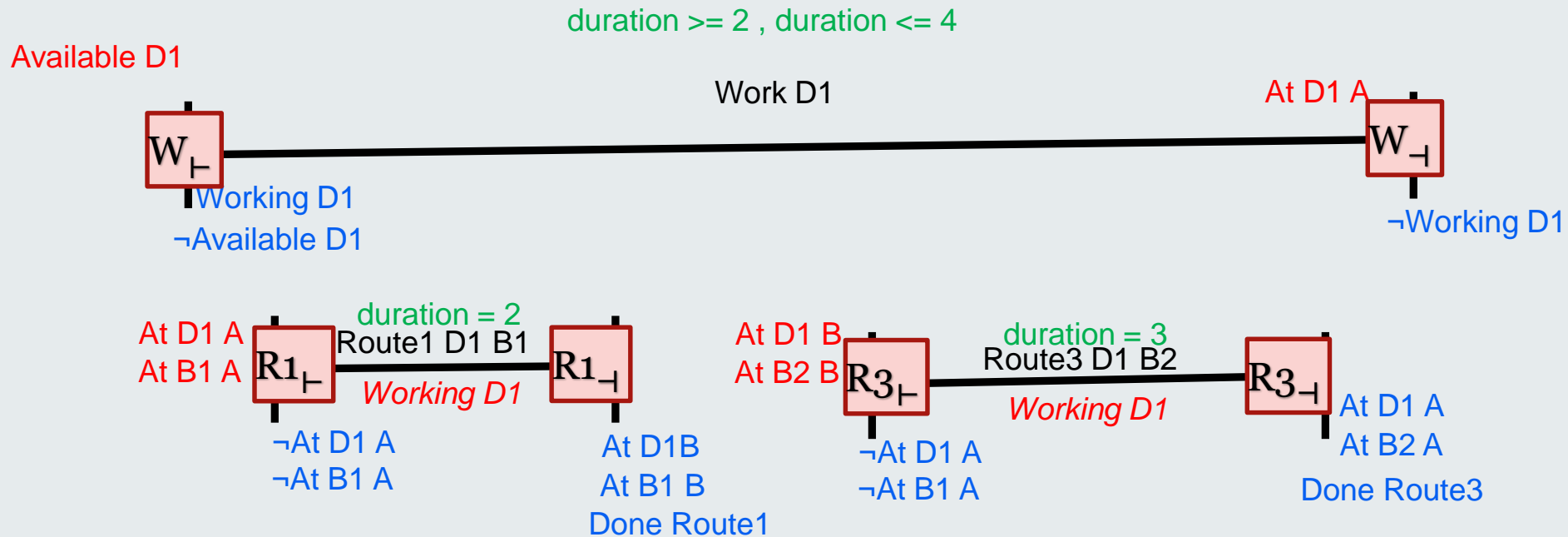
Invariant/overall conditions;

Durations constraints:

(= ?duration 4)

(and ( $\geq$  ?duration 2) ( $\leq$  ?duration 4))

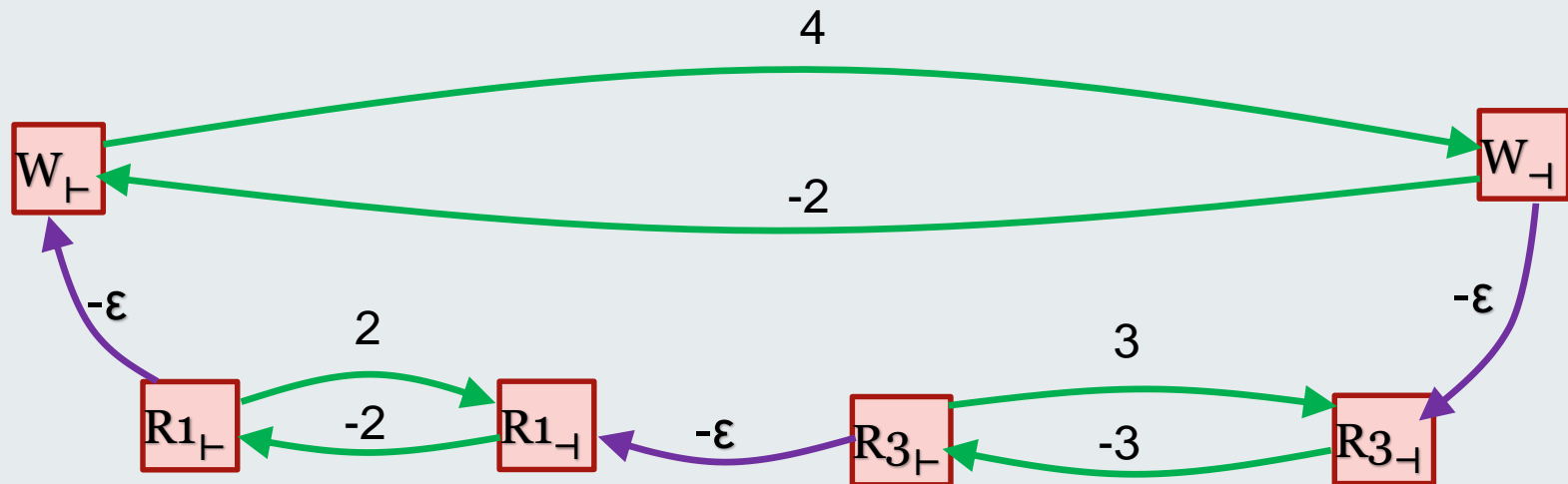
# Temporal Planning: Public Transport



## Three Challenges:

- Make sure ends can't be applied unless starts have.
- Overall Conditions.
- Duration constraints.

# Temporal Planning: Public Transport



## Constraints:

$$W_{t+1} - W_t \geq 2$$

$$W_{t+1} - W_t \leq 4$$

$$R1_t \geq W_t + \epsilon$$

$$R1_{t+1} - R1_t = 2$$

$$R3_t \geq R1_t + \epsilon$$

$$R3_{t+1} - R3_t = 3$$

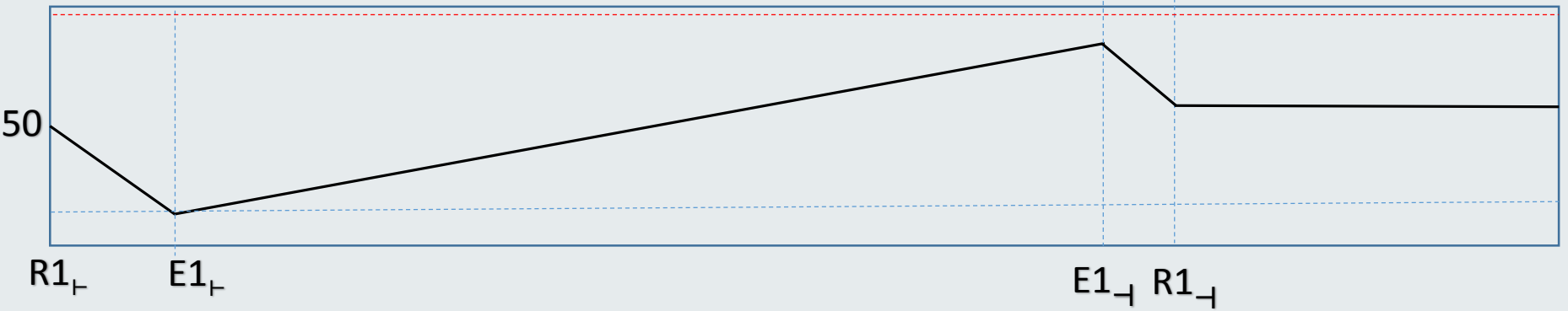
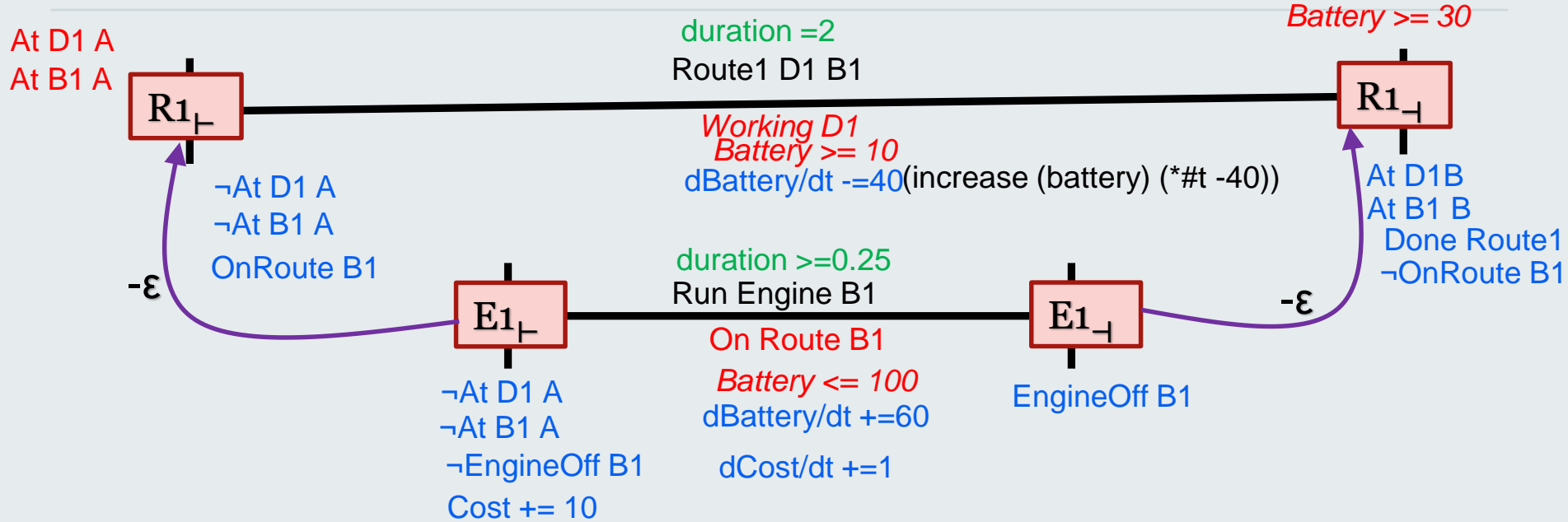
$$W_{t+1} \geq R3_{t+1} + \epsilon$$

# Continuous Linear Change



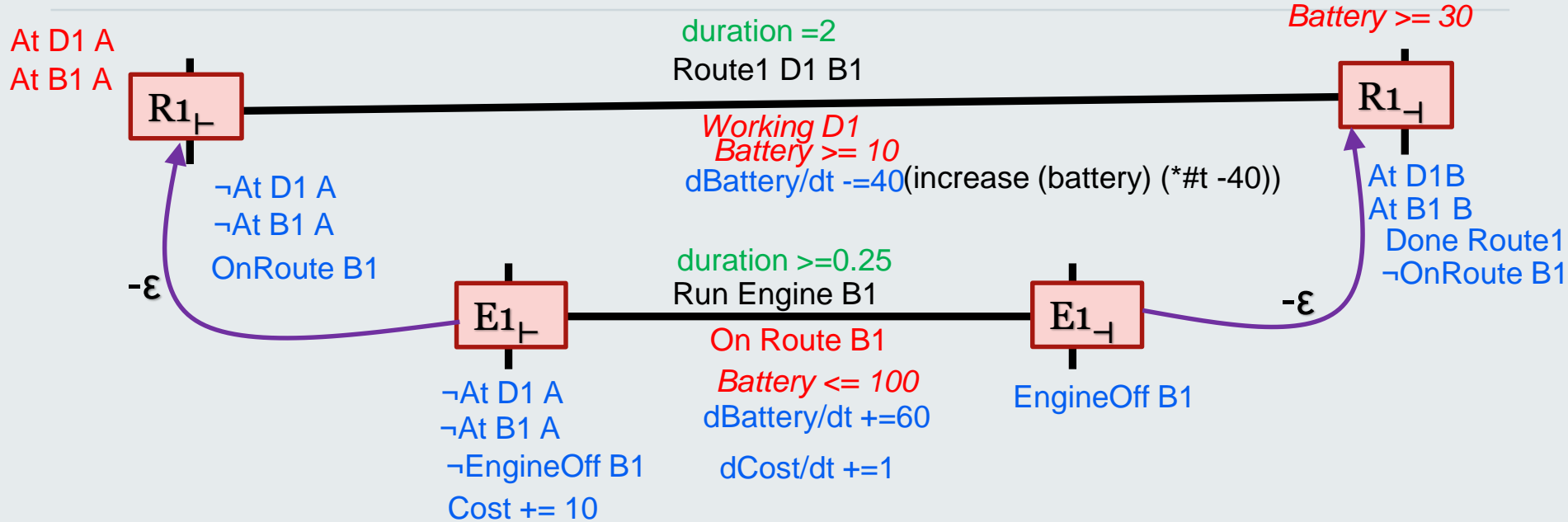
- Numeric quantities so far we have seen change instantaneously:
  - e.g. (at end (decrease (battery) 1))
  - **Or** (at end (decrease (battery) (+ (\*3 (?duration)) (\*0.5 (temperature))))
  - $V' = W \cdot V + C$
- In reality numeric values often change continuously, rather than discretely.
  - While the bus is running  $\frac{d \text{battery}}{dt} = -40$
  - e.g. (increase (battery) (\*#t -40))
  - Today we will deal with linear change only.

# Continuous Linear Change: Colin





# Continuous Linear Change: Colin



Temporal Constraints:

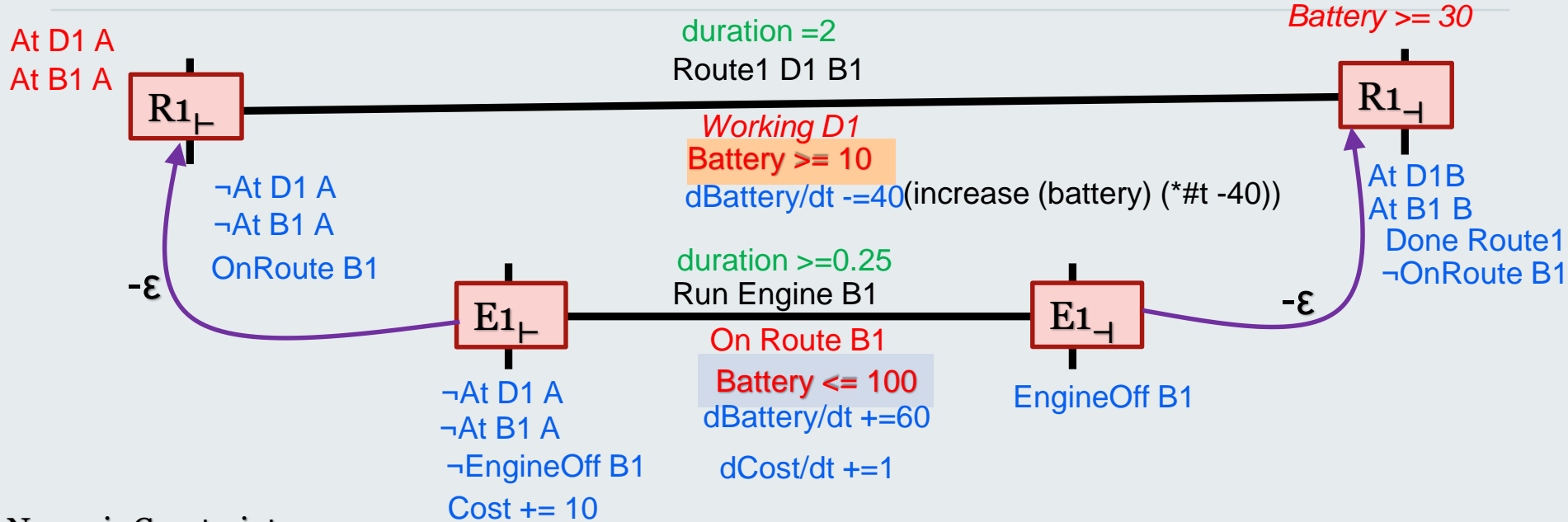
$$R1_{\perp} - R_{\perp} = 2$$

$$E1_{\perp} \geq W_{\perp} + \epsilon$$

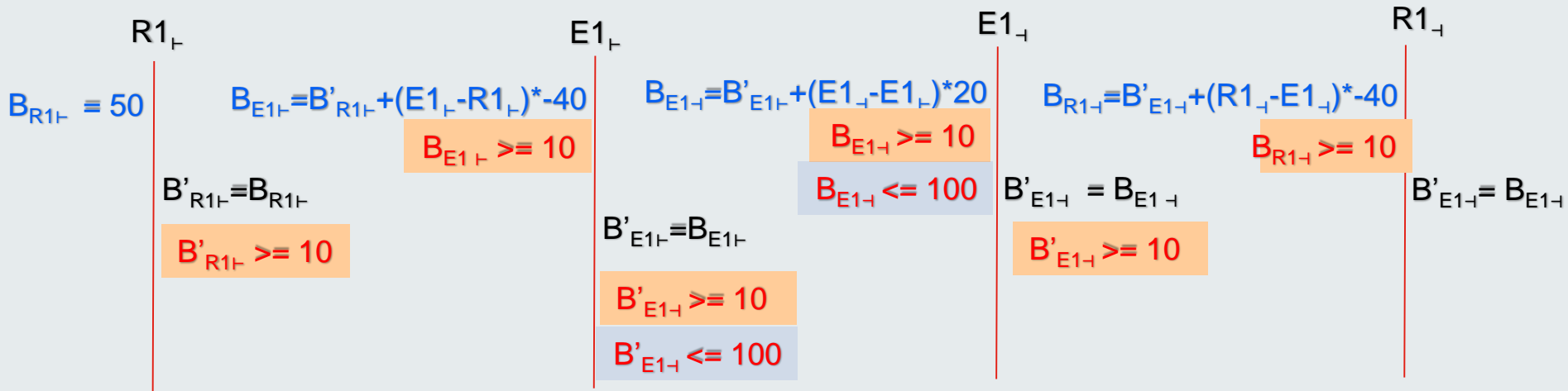
$$E1_{\perp} - E1_{\perp} \geq 0.25$$

$$R1_{\perp} \geq E1_{\perp} + \epsilon$$

# Continuous Linear Change: COLIN



Numeric Constraints:



# Writing The LP

---

- For each (snap) action,  $A_i$ , in the (partial) plan create the following LP variables for each numeric variable in the problem:
  - $v_i$ : the value of that variable immediately before  $A_i$  is executed;
  - $v'_i$ : the value  $v$  immediately after  $A_i$  is executed.
  - $\delta v_i$ : the rate of change active on  $v$  after  $A_i$  is executed.
- Create a single LP variable  $t_i$  to represent the time at which  $A_i$  will be executed.

# Writing the LP - Constraints

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- Initial values:
  - $v_0 =$  initial state value of  $v$ ;
- Temporal Constraints:
  - $t_i \geq t_{i-1} + \varepsilon$
  - $t_j - t_i \leq \max\_dur A$  (where  $t_j$  is the end of the action starting at  $t_i$ )
  - $t_j - t_i \geq \min\_dur A$  (where  $t_j$  is the end of the action starting at  $t_i$ )
- Continuous Change
  - $v_{i+1} = v'_i + \delta v_i (t_{i+1} - t_i)$
- Discrete Change:
  - $v'_i = v_i + \mathbf{w} \cdot \mathbf{v}_i$ ;
  - e.g. :  $v'_i = v_i + 2 u_i - 3w_i$

# Writing the LP - Constraints

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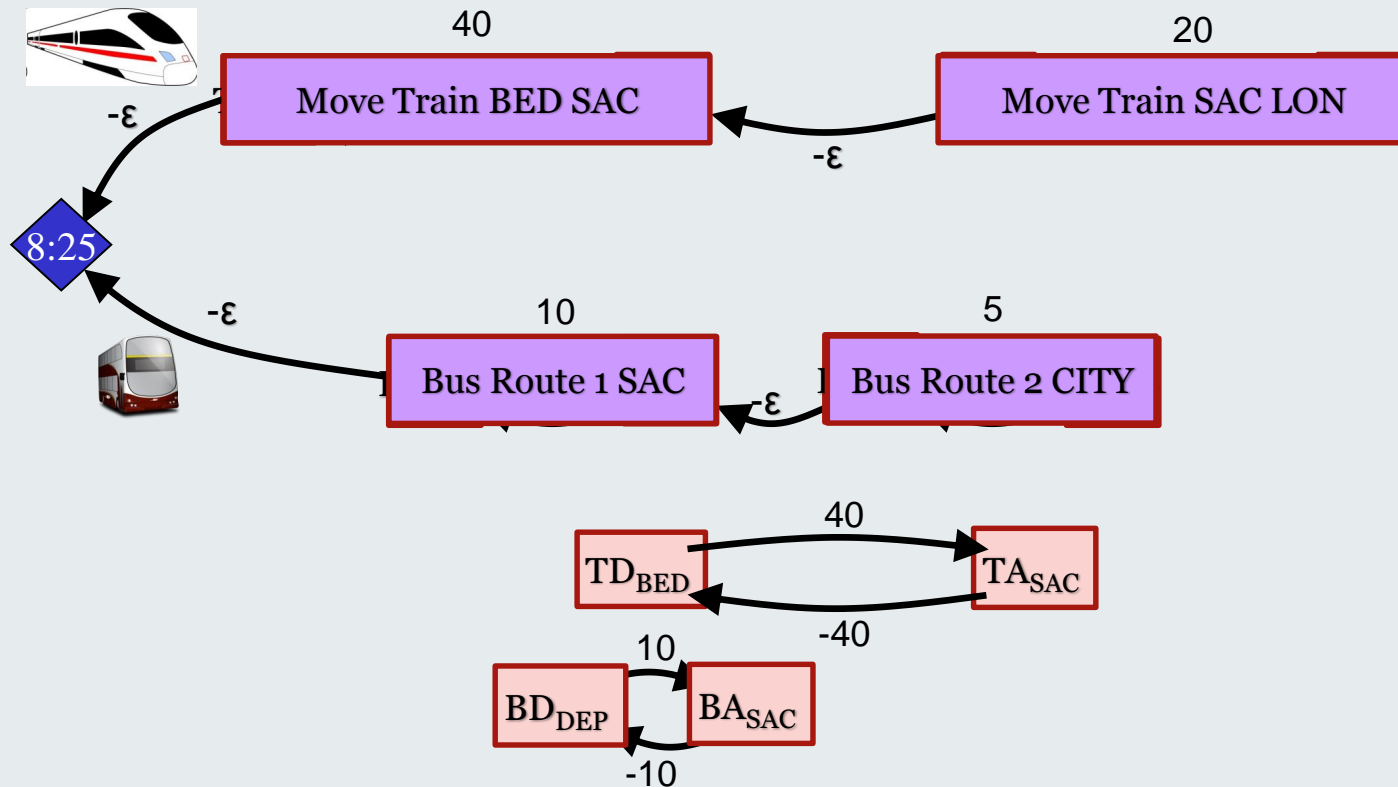
- Preconditions: constraints over  $v_i$ :
  - $\mathbf{w} \cdot \mathbf{v}_i \{>=, =, <= \} c$ ;
  - e.g.  $2w_i - 3u_i <= 4$ ;
- Invariants of A, must be checked before and after every step between the start (i) and end (j) of A.
  - $\mathbf{w} \cdot \mathbf{v}'_i \{>=, =, <= \} c$ ;
  - $\mathbf{w} \cdot \mathbf{v}_{i+1} \{>=, =, <= \} c$ ;
  - $\mathbf{w} \cdot \mathbf{v}'_{i+1} \{>=, =, <= \} c$ ;
  - ...
  - $\mathbf{w} \cdot \mathbf{v}_j \{>=, =, <= \} c$ ;

# Writing the LP - Notes

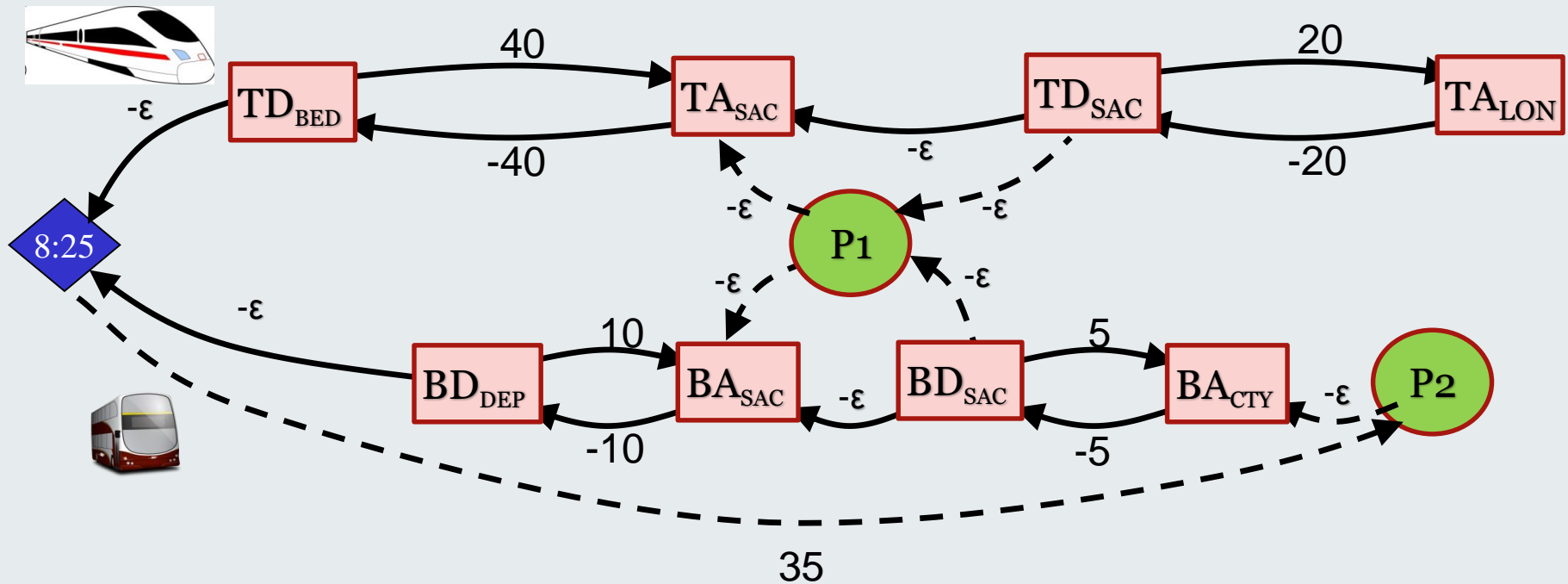
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- This only works for linear change
- Defining an objective function will ensure cost optimality for the given state only
- Action applicability:
  - To check if the next action is applicable we need the bounds on the current variables
  - Define  $t_{\text{now}}$  and  $v_{\text{now}}$ , for the next action, use LP to maximise and minimise  $v_{\text{now}}$

# Partial Order Planning Forwards: POPF



# Optimising Preferences: OPTIC and LPRPG-P



- The train and the bus are at the station simultaneously: (sometime (and (at train SAC) (at bus SAC)))
- The bus arrives in the city at 9am or earlier: (within 35 (at bus CITY)))



# Preferences

---

- Train arrives before bus departs:
  - $BD_{SAC} - TA_{SAC} \geq 0.01$
- Bus arrives before train departs:
  - $TD_{SAC} - BA_{SAC} \geq 0.01$
- Bus arrives at CTY by 9am (time 35):
  - $BA_{CTY} \leq 35$
  
- But these are not hard constraints
  
- Use Big M constraints

# Big M Constraints

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- We need:
  - A 0/1 integer variable per preference  $p_1, p_2$
  - A very large constant  $M$ .
- Train arrives before bus departs:
  - $BD_{SAC} - TA_{SAC} + Mp_1 \geq 0.01$
- Bus arrives before train departs:
  - $TD_{SAC} - BA_{SAC} + Mp_1 \geq 0.01$
- Bus arrives at CTY by 9am (time 35):
  - $BA_{CTY} - Mp_2 \leq 35$
- In the objective function:
  - Minimise: whatever +  $5 p_1 + 2 p_2$

# Relationship Between Planners

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- $\text{CRIKEY} = \text{FF} + \text{STN}$ ;
- $\text{Colin} = \text{CRIKEY} \text{ s/STN/LP/}$ ;
- $\text{POPF} = \text{COLIN} + \text{Fewer ordering constraints}$ ;
- $\text{OPTIC} = \text{POPF} + \text{Preferences}$ .

# Planners Performance

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- Heuristic computation is notoriously expensive:
  - An analysis showed that FF spends ~80% of its time evaluating the heuristic.
- COLIN:
  - Empirically using an STP scheduler scheduling accounts for on average less than 5% of state evaluation time.
  - For CLP and CPLEX (LP solvers) the figures are 13% and 18% respectively.
  - So better than calculating the heuristic.

# Planners Performance Cont.

- OPTIC

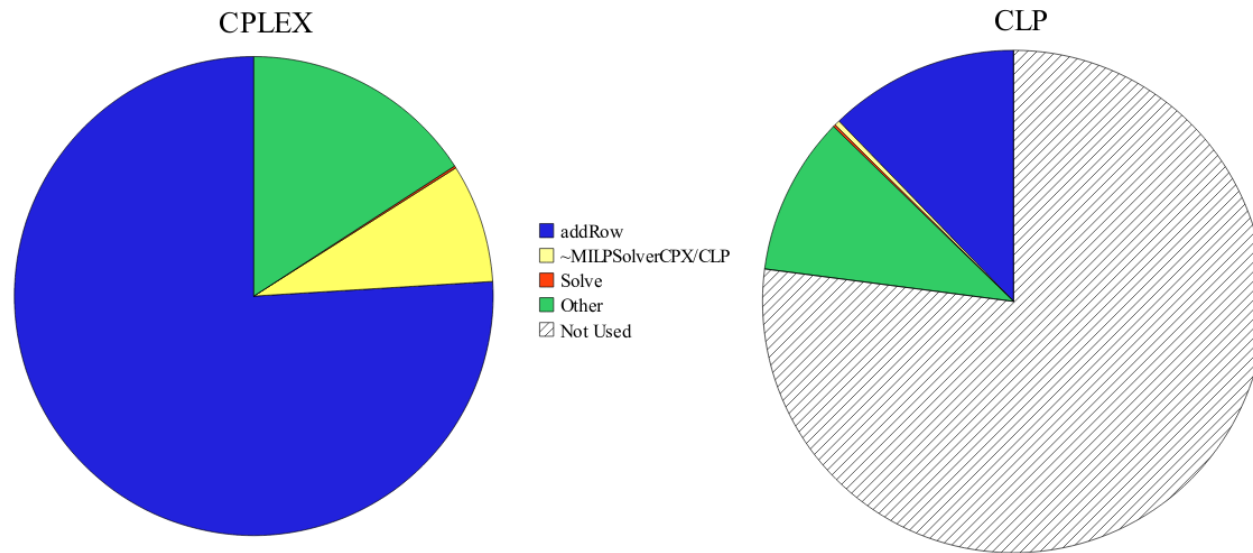


Figure 20: Time spent in various activities by each of the solvers, CPLEX and CLP, viewed as a proportion of the total time spent by CPLEX. The slice labelled ‘~MILPSolverCPX/CLP’ is time spent in the destructor for the MILP solver in CPLEX or CLP: this is a house-keeping operation in the implementations (which are both written in C++).

**Questions ?**